

# The Crystal Structure of Magnesium Chloride Dodecahydrate, $\text{MgCl}_2 \cdot 12\text{H}_2\text{O}$

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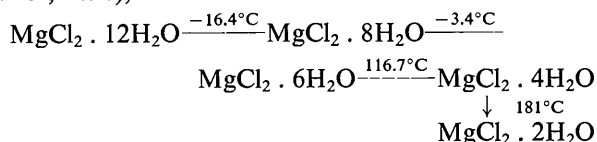
Magnesium chloride dodecahydrate crystallizes in the monoclinic space group  $P2_1/c$  with unit-cell dimensions  $a = 8.59$ ,  $b = 14.40$ ,  $c = 8.75$  Å,  $\beta = 129.6^\circ$ ,  $D_x = 1.241$  g.cm $^{-3}$ ,  $Z = 2$ .

The phase problem was solved by the direct method. The positions of all atoms except the hydrogen atoms were deduced from the three-dimensional Fourier synthesis, and refined by anisotropic least squares. All the hydrogen atoms are involved in hydrogen bonding and their approximate positions could be deduced from geometrical considerations.

The structure consists of  $[\text{Mg}^+(\text{H}_2\text{O})_6]$  and  $[\text{Cl}^-(\text{H}_2\text{O})_6]$  octahedra which are corner-linked in layers and hydrogen-bonded together between these layers. The ionic-bonded cationic octahedra are regular, while the larger hydrogen-bonded anion octahedra are considerably distorted.

## Introduction

Magnesium chloride forms five hydrates, stable over the following temperature ranges, (van't Hoff & Meyerhoffer, 1898),



Only the structure of the room temperature form has been determined (Andress & Gundermann, 1934). The octahydrate is reported to be dimorphic (van't Hoff & Meyerhoffer, 1898). No hydrate is reported for  $\text{MgF}_2$ , but  $\text{MgBr}_2$  forms hexa- and deca-hydrates, and  $\text{MgI}_2$  octa- and deca-hydrates (Panfilov, 1894). In this work, we have determined the structure of  $\text{MgCl}_2 \cdot 12\text{H}_2\text{O}$  as part of a program for investigating the structural behavior of ions and molecules in the presence of a majority of water molecules.

## Experimental

The crystals of  $\text{MgCl}_2 \cdot 12\text{H}_2\text{O}$  were obtained from a stoichiometric aqueous solution by cooling to  $-20^\circ$  to  $-25^\circ\text{C}$  in a bromobenzene bath, (van't Hoff & Meyer-

hoffer, 1898). Nucleation was promoted by wetting a thread with solution, pre-cooling it in dry ice and immersing into the cool solution. The well-developed crystals of  $\sim 0.3$  mm diameter, which formed on the thread, were separated and inserted into capillary tubes in a cold-box at  $-35^\circ\text{C}$ . The X-ray data were obtained with  $\text{Cu } K\alpha$  radiation and a Weissenberg camera operating in the cold-box, as described by Allen, Jeffrey & McMullan (1963).

The crystal data are as follows,

Monoclinic, m.p.  $-16.4^\circ\text{C}$

$a = 8.59 \pm 0.05$ ,  $b = 14.40 \pm 0.03$ ,  $c = 8.75 \pm 0.05$  Å,

$\beta = 129.6 \pm 0.2^\circ$ ,  $D_x = 1.241$  g.cm $^{-3}$ ,  $Z = 2$ ,

$\mu(\text{Cu } K\alpha) = 42.58$  cm $^{-1}$ .

The space group is  $P2_1/c$ , from systematic extinctions  $h0l$  absent for  $l$  odd,  $0k0$  absent for  $k$  odd. Hence the formula unit is centrosymmetrical, and the Mg ions are on the twofold centers of symmetry.

The intensities were recorded by the multiple-film technique on 0 to 5, 0 to 6, and 0 and 1 layers about the  $a$ ,  $b$  and  $c$  axes respectively. The intensities were eye-estimated, correlated and  $L_p$ -corrected with the use of IBM 1620 programs. Of the 2260 possible reflections within the  $\text{Cu } K\alpha$  sphere, 1609 were recorded. Of these, 206 were not observed and were given values of zero. The approximate absolute scale and mean isotropic temperature factors were determined from a Wilson plot.

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Table 1. Structural parameters for  $\text{MgCl}_2 \cdot 12\text{H}_2\text{O}$ , with the standard deviations referred to the last significant digits in parentheses

The  $b_{ij}$  are defined by:  $T = \exp \{ - [10^{-4}(b_{11}h^2 + b_{22}k^2 + b_{33}l^2 + 2b_{12}hk + 2b_{13}hl + 2b_{23}kl)] \}$ .

| Atom | $x$         | $y$         | $z$         | $b_{11}$ | $b_{22}$ | $b_{33}$ | $b_{12}$ | $b_{13}$ | $b_{23}$ |
|------|-------------|-------------|-------------|----------|----------|----------|----------|----------|----------|
| Mg   | 0           | 0           | 0           | 38 (6)   | 11 (1)   | 57 (6)   | —        | 33 (5)   | —        |
| Cl   | 0.2628 (3)  | 0.3100 (1)  | 0.2505 (3)  | 102 (4)  | 20 (1)   | 98 (4)   | —1 (2)   | 65 (3)   | —2 (1)   |
| O(1) | 0.2390 (8)  | 0.0493 (4)  | 0.0225 (9)  | 74 (11)  | 33 (3)   | 148 (13) | —2 (5)   | 78 (10)  | 27 (5)   |
| O(2) | —0.0082 (9) | 0.1222 (4)  | 0.1175 (9)  | 170 (14) | 18 (3)   | 175 (14) | —10 (5)  | 145 (12) | —15 (5)  |
| O(3) | 0.2060 (9)  | —0.0565 (4) | 0.2798 (8)  | 122 (12) | 23 (3)   | 80 (11)  | 18 (5)   | 55 (10)  | 15 (4)   |
| O(4) | 0.4357 (10) | —0.2104 (5) | 0.3427 (10) | 135 (13) | 28 (3)   | 141 (14) | 14 (6)   | 84 (12)  | 24 (5)   |
| O(5) | 0.3518 (9)  | —0.4889 (4) | 0.1626 (8)  | 107 (12) | 28 (3)   | 105 (11) | —5 (5)   | 63 (10)  | —1 (5)   |
| O(6) | 0.1475 (9)  | —0.3482 (5) | 0.1812 (9)  | 103 (12) | 36 (3)   | 127 (13) | 10 (6)   | 72 (11)  | 17 (5)   |

Table 2. *Observed and calculated structure factors*

|             |            |            |            |            |            |            |             |
|-------------|------------|------------|------------|------------|------------|------------|-------------|
| K= 0 H= 0   | 1 235 247- | 2 207 216  | 1 79 52-   | K= 8 H= 3  | K= 14 H= 4 | 1 123 120  | 1 81 65-    |
| 2 53+ 712-  | 2 112 96   | 4 138 139  | 2 84 50-   | 0 335 335  | 0 102 88-  | K= 8 H= 6  | 2 99 76     |
| 4 331 362   | 3 193 247  | K= 11 H= 1 | 3 80+ 66   | 1 127 120  | 1 106 106- | 0 260 347  | 3 93 84     |
| 6 80 57     | 4 69 56-   | 0 162 149- | 4 269 308  | 2 80+ 48   | 2 99 102   | K= 9 H= 2  | 4 308 316   |
| 8 173 192   | 5 87 84-   | 1 323 382  | 6 114 133- | K= 9 H= 3  | K= 15 H= 4 | 1 173 228  | 5 204 189   |
| K= 1 H= 0   | K= 15 H= 5 | 2 139 137  | K= 9 H= 2  | 0 216 207- | 1 121 146  | K= 11 H= 6 | 6 210 225-  |
| 1 325 368   | 3 73 62    | 3 125 120- | 0 52+ 9    | 1 98 63    | K= 16 H= 4 | 1 52 56    | 7 107 107-  |
| 2 279 265-  | 2 109 104  | 4 150 150- | 1 230 216  | 2 167 163  | 0 170 235  | K= 12 H= 6 | 8 142 149   |
| 3 561+ 673  | 3 102 92   | 5 229 243  | 3 161 153  | 3 122 116  | K= 0 H= 5  | 0 104 137  | K= 8 H= -1  |
| 5 53 38-    | 4 59 34-   | 4 102 107  | 4 87 58-   | 4 115 116- | 0 277 265  | 0 207 235  | 1 242 235   |
| 6 62 50     | K= 16 H= 0 | K= 17 H= 1 | 5 68 61    | K= 10 H= 3 | 2 262 269  | 0 157 176  | 2 254 286   |
| 7 126 119   | 0 269 287  | 0 97 64    | 6 48 42    | 0 309 97-  | 4 95 97-   | K= 1 H= 7  | 3 65+ 13-   |
| K= 2 H= 0   | 2 83 88-   | 1 71 60    | K= 10 H= 2 | 1 126 120- | K= 1 H= 5  | 0 177 191  | 4 176 176-  |
| 0 176 122   | 5 136 176  | 4 160 175  | 0 151 145  | K= 11 H= 3 | 0 253 235- | 1 47 32    | 5 62+ 43    |
| 1 214 233   | K= 17 H= 0 | K= 13 H= 1 | 1 211 201  | 0 181 188  | 1 122 80   | 2 113 112  | 6 62+ 61    |
| 2 443+ 596  | 1 412 117  | 0 77+ 55+  | 2 305 305  | 2 131 159- | 2 131 97   | K= 2 H= 7  | 8 66 66     |
| 3 428+ 503- | K= 18 H= 0 | 1 122 98-  | 3 238 257- | 2 131 115- | 3 57 40    | 0 144 148- | K= 9 H= -1  |
| 4 66 65     | 2 94 13    | 2 87 83    | 4 78 58-   | 3 250 283  | 4 126 121- | K= 3 H= 7  | 1 271 261   |
| 5 236 244   | K= 0 H= 1  | 3 233 277  | 5 114 113  | K= 12 H= 3 | K= 2 H= 5  | 1 190 204  | 2 237 240   |
| 6 219 257   | 0 330 267  | 4 50+ 6    | K= 11 H= 2 | 0 93 74    | 0 274 259  | K= 4 H= 7  | 3 175 156-  |
| 7 139 159-  | 2 71 62    | 5 75 84-   | 0 176 169- | 1 64+ 25-  | 1 90 52-   | 0 60 29-   | 4 183 173-  |
| K= 3 H= 0   | 4 154 175  | K= 14 H= 1 | 2 60+ 38-  | 3 93 81-   | 2 61+ 1    | 1 70 59-   | 5 109 91    |
| 1 261 252   | 5 37+ 24   | 0 61+ 34   | 3 151 147  | 4 71 63    | 3 90 66-   | K= 5 H= 7  | 6 192 205   |
| 2 80 50-    | K= 1 H= 1  | 1 96 83    | 4 63+ 53   | K= 13 H= 3 | 4 118 112  | 0 33+ 5    | 7 50+ 42    |
| 3 258 264   | 0 242 267- | 2 58+ 56   | 5 111 111  | 1 187 222  | K= 3 H= 5  | 1 71 59-   | 8 86 96-    |
| 4 107 110-  | 1 547 607  | 3 74 99    | K= 12 H= 2 | 3 71 54-   | 0 69 52-   | K= 6 H= 7  | K= 10 H= -1 |
| 5 190 198   | 2 253 256  | 4 117 113  | 0 104 76   | K= 14 H= 3 | 1 117 80-  | 0 140 135  | 1 73 52-    |
| 6 75 68-    | 4 81 72-   | K= 15 H= 1 | 1 130 126- | 2 125 117  | 2 147 134- | 1 109 119  | 2 200 205   |
| 7 95 95     | 4 404 414- | 0 225 224  | 2 119 125  | K= 15 H= 3 | 3 219 209  | K= 7 H= 7  | 3 129 113-  |
| 8 52 40     | 5 52 33-   | 1 159 190  | 4 147 142  | 0 102 84-  | 4 65 57    | 0 137 161  | 4 92 63     |
| K= 4 H= 0   | 4 224 224  | 2 42+ 32-  | 5 71 78-   | 2 73 66    | K= 4 H= 5  | 1 122 173  | 7 73 70     |
| 0 255 268   | 2 104 73   | 3 45+ 44-  | K= 13 H= 2 | 0 108 87   | 0 108 87   | K= 9 H= 7  | K= 11 H= -1 |
| 1 349+ 428- | K= 1 H= 1  | 4 76 64    | 0 86 40-   | 0 132 146  | 1 95 75    | 0 87 98-   | 1 234 232-  |
| 7 126 116   | 0 333 316  | K= 16 H= 1 | 1 121 128  | 1 52 44-   | 2 116 96   | K= 10 H= 7 | 2 285 275-  |
| 8 113 93    | 1 614 622- | 0 101 109  | 2 80 47    | K= 17 H= 3 | 3 177 158- | 0 93 130   | 3 280 298   |
| 4 199 212   | 5 177 14   | 2 82 71    | K= 14 H= 2 | 0 106 115  | 4 64 47    | K= 0 H= 8  | 4 188 185   |
| 5 147 165-  | 4 163 109  | 3 62 57    | 0 168 147  | K= 0 H= 4  | K= 5 H= 5  | 0 190 216  | 5 96 68-    |
| 4 128 158   | 5 84 50-   | K= 17 H= 1 | 1 193 237  | 0 534 526  | 0 74 58    | K= 1 H= 8  | 6 107 102-  |
| 7 82 119    | 4 117 92   | 0 127 123- | 3 142 155- | 2 105 72-  | 1 128 117  | 0 39 40-   | 7 146 137   |
| 8 47 51     | 7 31+ 18   | 1 67 58    | 4 115 141  | 4 288 351  | 2 152 151- | K= 2 H= 8  | K= 12 H= -1 |
| K= 5 H= 0   | K= 3 H= 1  | 2 114 121  | K= 15 H= 2 | K= 1 H= 4  | 3 53 44-   | 0 58 40    | 2 158 149   |
| 1 208 159-  | 0 301 292  | K= 18 H= 1 | 0 80 66-   | 0 305 285- | 4 44 41    | K= 3 H= 8  | 5 123 107   |
| 2 135 131-  | 1 432 424  | 0 75 72    | 1 51 58    | 1 51 508   | K= 7 H= 5  | 0 53 38    | 6 84 97     |
| 3 379+ 480  | 7 63+ 77-  | 1 40 42    | 2 45+ 55   | 2 81 54    | 0 254 273- | K= 4 H= 8  | 7 85 96     |
| 4 120 145   | 3 46+ 490  | K= 0 H= 2  | 3 95 97    | 5 104 100  | K= 6 H= 5  | 0 53 39    | K= 13 H= -1 |
| 5 110 119   | 4 204 164  | 0 525 540- | K= 16 H= 2 | K= 2 H= 4  | 0 54+ 16-  | K= 5 H= 8  | 1 187 173   |
| 6 140 166-  | 5 149 114  | 2 448 454  | 0 93 90-   | 1 307 306  | 1 96 66-   | 0 35 27-   | 2 175 169-  |
| K= 6 H= 0   | 6 127 92   | 4 335 318- | 1 46+ 43   | 2 318 304  | 2 215 223  | K= 0 H= -1 | 3 87 67-    |
| 0 474+ 594  | 7 192 178  | 6 182 210  | 2 209 266  | 3 273 271- | 3 59 46    | 2 409+ 501 | 4 118 104   |
| 1 544+ 771  | K= 4 H= 1  | K= 1 H= 2  | K= 17 H= 2 | 4 72 53    | K= 7 H= 5  | 4 146 162  | 5 241 285   |
| 2 171 182-  | 3 484 422- | 0 187 138  | 1 52 17    | 5 108 105  | 2 120 123  | 6 36+ 26   | 6 51+ 49-   |
| 3 239 279-  | 1 183 130  | 2 340 305- | K= 0 H= 3  | K= 3 H= 4  | 3 109 98   | 8 213 195  | K= 14 H= -1 |
| 4 143 169   | 2 282 266  | 3 100 48-  | 0 239 196  | 0 113 76   | K= 8 H= 5  | K= 1 H= -1 | 1 66+ 31    |
| 5 86 72     | 3 195 168- | 4 146 119  | 2 215 203  | 1 172 141  | 0 149 140  | 3 117 115  | 2 130 105   |
| 6 60 55     | 4 276 227  | 5 87 62    | 4 90 96    | 2 103 81-  | 1 60+ 38   | 4 237 219  | 4 95 100    |
| 7 151 158-  | 5 147 114- | 7 149 16   | K= 1 H= 3  | 3 169 156  | 2 110 83   | 5 202 176  | 5 89 69-    |
| 8 85 97     | 6 61 30-   | 8+ 2 H= 2  | 0 548 626  | 5 56 29    | 3 110 111- | 6 226 233- | K= 15 H= -1 |
| K= 7 H= 0   | 7 65 39    | 0 112 28   | 1 239 219  | K= 4 H= 4  | K= 9 H= 5  | 7 195 193  | 2 182 189   |
| 1 131 115   | K= 5 H= 1  | 1 440 430- | 2 331 334- | 0 93 70    | 0 97 64    | 8 222 231  | 3 101 102   |
| 2 131 123   | 0 66 50    | 2 225 183  | 3 146 126  | 1 281 268- | 1 58+ 30   | 9 26+ 14   | 4 132 132-  |
| 3 81 84     | 1 613+ 719 | 3 256 250  | 4 180 150  | 3 227 198  | 2 136 133- | K= 2 H= -1 | K= 16 H= -1 |
| 4 70 45-    | 2 335 330- | 4 95 61    | 5 74 50    | 4 211 197  | 3 62 56    | 1 93 86-   | 1 52+ 21-   |
| 6 83 72-    | 3 335 294- | 5 217 200- | 6 71 68-   | 5 109 110- | K= 10 H= 5 | 2 283 335  | 2 80 74     |
| K= 8 H= 0   | 4 162 136  | 6 88 78-   | K= 2 H= 3  | K= 5 H= 4  | 2 137 150  | 3 326 335- | 3 63 54     |
| 0 255 207-  | 5 345 342  | 7 109 165  | 0 317 244  | 0 42+ 6-   | K= 11 H= 5 | 4 217 211  | 4 48+ 43    |
| 1 289 282-  | 6 175 166- | K= 3 H= 2  | 1 47 29    | 1 239 258  | 1 149 177  | 5 306 314  | K= 17 H= -1 |
| 2 381 433   | 7 47 35-   | 0 228 193  | 4 81 57    | 2 107 72   | 6 53 35    | 8 185 199  | 2 121 102   |
| 4 300 344-  | K= 6 H= 1  | 1 364 330- | 5 81 37    | 3 37+ 18   | 1 88 87-   | 6 185 199  | 1 210 230-  |
| 5 75+ 69    | 0 13+ 86   | 7 103 71-  | 6 136 145  | 4 49 33-   | K= 13 H= 5 | K= 3 H= -1 | K= 0 H= -2  |
| 6 200 283   | 1 173 147  | 4 90 55-   | 0 222 186- | 5 64 71    | 0 88 71-   | 1 477+ 742 | 2 518+ 829  |
| 7 63 62-    | 2 69+ 7-   | 6 94 92    | 1 338 332  | 0 113 83   | 1 107 125- | 2 248 274  | 4 685+ 774- |
| K= 9 H= 0   | 3 87 53    | 7 48 41-   | 2 240 210  | 0 87 96    | K= 14 H= 5 | 3 253 332- | 6 320 345   |
| 1 70 54     | 4 282 269  | K= 4 H= 2  | 3 90 67    | 2 55+ 20   | 0 80 42-   | 4 92 82-   | 8 58 33-    |
| 2 120 168   | 5 131 95-  | 0 155 95-  | 4 94 68-   | 3 166 158- | K= 0 H= 6  | 5 458+ 621 | K= 1 H= -2  |
| 3 57+ 29    | 6 71 52    | 1 144 118  | 5 173 177  | 4 156 140  | 0 116 128  | 6 191 209  | 1 288 347   |
| 4 133 119-  | 7 35+ 11-  | 2 271 229  | 6 106 104  | K= 7 H= 4  | K= 1 H= 6  | 7 105 92-  | 2 190 148-  |
| 5 86 64     | K= 7 H= 1  | 3 206 161- | K= 4 H= 3  | 0 155 118- | 0 55 40    | 8 70 63-   | 3 413 362-  |
| K= 10 H= 0  | 0 258 247- | 4 53+ 12   | 0 245 216  | 2 111 83   | 1 60 41-   | 9 73 109   | 5 131 104   |
| 0 265 239   | 1 325 348  | 5 218 214  | 1 46+ 43-  | K= 8 H= 4  | 2 40+ 29-  | K= 4 H= -1 | 6 184 173   |
| 1 255 266-  | 2 198 148  | 6 103 93   | 2 74 47    | 0 241 219- | 3 109 113  | 1 482+ 567 | 7 51 21     |
| 2 153 164-  | 3 239 238  | K= 5 H= 2  | 3 127 99   | 1 82 61    | K= 2 H= 6  | 2 304 335  | 8 37+ 21-   |
| 3 119 102   | 4 246 224- | 0 260 243- | 4 104 75   | 2 280 298  | 0 260 252  | 3 107 84   | 9 138 123   |
| 4 106 93    | 5 88 79-   | 1 73 41-   | 5 54 22    | 4 128 128- | 1 175 171- | 4 345 434  | K= 2 H= -2  |
| 5 93 89-    | 6 98 99    | 2 205 182  | 6 100 104  | K= 9 H= 4  | 2 134 91-  | 5 55+ 42   | 1 440 480-  |
| 7 102 121   | 7 124 155  | 3 233 208  | K= 5 H= 3  | 0 159 140  | 3 50 44    | 8 76 92    | 2 537+ 629- |
| K= 11 H= 0  | K= 6 H= 1  | 4 54 48    | 0 108 62-  | 1 117 101  | K= 3 H= 6  | K= 5 H= -1 | 3 191 210   |
| 2 126 104-  | 0 284 271  | 5 103 85   | 1 328 311- | 2 133 137- | 0 44+ 22   | 1 331 366- | 4 609 653   |
| 5 105 112   | 1 77 42-   | 0 108 62-  | 2 387 406  | 4 53 56    | 1 96 94    | 2 431+ 470 | 5 105 69-   |
| 6 79 64     | 2 77 41    | K= 6 H= 2  | 3 349 354  | K= 10 H= 4 | 2 63 41-   | 3 342 393  | 6 198 181-  |
| K= 12 H= 3  | 3 77 61    | 1 208 93-  | 4 136 130- | 0 200 182  | K= 4 H= 6  | 4 43+ 25   | 7 219 220   |
| 0 136 118-  | 4 155 157  | 2 334 344  | 5 68 60-   | 1 112 117- | 1 142 142  | 5 48 33-   | 8 142 138   |
| 1 170 147   | 5 136 176- | 3 465 227  | K= 6 H= 3  | 2 149 148- | 3 70 62-   | 6 94 78    | K= 3 H= -2  |
| 2 122 88    | 6 62+ 32   | 4 55+ 23   | 0 274 243  | 3 71 71    | K= 5 H= 6  | 7 211 268  | 1 461 492   |
| 3 151 142-  | K= 4 H= 1  | 5 180 170- | 3 171 132  | K= 11 H= 4 | 0 38+ 35   | 8 56 59-   | 2 186 182   |
| 4 73+ 60    | 0 369 351  | 6 111 87   | 4 140 120  | 2 66+ 33-  | 1 50 31    | 2 337 390  | 5 82 63-    |
| 5 133 174   | 1 286 313  | K= 7 H= 2  | 5 100 96   | K= 12 H= 4 | 2 46 30-   | 3 87 82    | 6 150 136   |
| 6 75 72     | 2 375 430- | 0 71 36    | K= 7 H= 3  | 0 118 106- | K= 6 H= 6  | 4 87 75-   | 7 198 206   |
| K= 13 H= 0  | 4 171 151  | 1 338 337  | 0 320 323  | 1 113 102  | 0 83 63    | 5 117 108  | 9 32+ 8     |
| 2 165 158   | 5 117 105  | 4 105 81   | 1 325 335  | 2 131 113  | 1 141 142- | 6 137 131  | K= 4 H= -2  |
| 3 139 146   | 6 76 80-   | 5 123 135  | 3 66+ 47   | 3 55 51-   | 2 137 114  | 7 56 34-   | 1 188 229-  |
| 4 67+ 54-   | K= 13 H= 1 | K= 8 H= 2  | 4 143 141  | K= 13 H= 4 | 0 73+ 72   | 2 129 103- |             |
| K= 14 H= 0  | 0 127 84-  | K= 8 H= 2  | 5 47+ 19   | 0 94 79    |            |            |             |
| 0 88 73     | 1 115 110- | 0 566+ 689 |            |            |            |            |             |

\* Unobserved or very weak reflections omitted from final refinement calculations.

+ Reflections omitted because of extinction.

Table 2 (cont.)

A large table containing numerical data arranged in columns, representing Table 2 (continued). Each row contains multiple columns of numbers and associated symbols like K, H, M, and their values.

### Determination and refinement of the structure

The phase problem was solved directly with the three-dimensional sign-correlation IBM 1620 program of Beurskens (1964). The first application of this method gave 284 signs, all of which were for the reflections  $h+k=2n$ . The resulting  $E$ -map showed twice the proper number of atomic peaks and a false twofold symmetry axis at  $\frac{1}{2}$ ,  $y$ ,  $\frac{1}{2}$ . A sign determination of the weaker reflections of the type  $h+k=2n+1$  gave 47 assignments of high probability but low frequency. Inclusion of these in the  $E$ -map increased some peaks and diminished others, thereby permitting an identification of the asymmetric unit of structure. The coordinates of all the atoms in the general positions, except the hydrogen atoms, were derived and the initial isotropic structure factor calculations gave an agreement index of  $R=0.24$ . For the calculation of structure factors, the atomic scattering factors of magnesium and chlorine ions, and oxygen atoms were taken from *International Tables for X-Ray Crystallography*.

The structure refinement was carried out with the full-matrix least-squares IBM 7090 program of Busing & Levy (1959). The function minimized was  $\Phi = \sum_{hkl} \omega(|F_{\text{obs}}| - G|F_{\text{calc}}|)^2$ , where  $G$  is the overall scale factor which varies between cycles and  $\omega = 1/(a + b|F_{\text{obs}}| + c|F_{\text{obs}}|^2)$  with  $a=4$ ,  $b=1$ ,  $c=0.029$ . Four cycles in the isotropic mode reduced  $R$  to 0.13, and three anisotropic cycles gave a further small reduction to 0.11. The hydrogen atoms could not be directly located, since the three-dimensional difference Fourier synthesis showed twice as many weak maxima as hydrogen atoms. Although it was possible to predict the donor/acceptor positions of all the hydrogen atoms in hydrogen bonds, the inclusion of these atoms had no effect on the agreement indices and therefore they were not included in the final refinement calculations. The standard deviations quoted in Table 1 are directly from the least-squares program output and take no account of the omission of the hydrogen scattering.

The atomic parameters are given in Table 1. The structure factors are given in Table 2. Those marked with asterisks were omitted from the final refinement

calculation because they were suspected to be unreliable, either by reason of extinction, or because they corresponded to unobserved reflections or to very weak spots on the photographs. The ionic and hydrogen bond angles are given in Tables 3, 4 and 5.

Table 4. *Hydrogen bond distances and angles within the  $\text{Cl}(\text{H}_2\text{O})_6$  octahedron*

| From atoms<br>in positions | to<br>atoms | in positions                        | Distance    |
|----------------------------|-------------|-------------------------------------|-------------|
| Cl                         | O(1)        | $x, \frac{1}{2}-y, \frac{1}{2}+z$   | 3.230 (7) Å |
| $(x, y, z)$                | O(5)        | $x, y, z$                           | 3.211 (7)   |
|                            | O(4)        | $1-x, \frac{1}{2}+y, \frac{1}{2}-z$ | 3.186 (8)   |
|                            | O(2)        | $x, y, z$                           | 3.261 (8)   |
|                            | O'(4)       | $1-x, -y, 1-z$                      | 3.112 (8)   |
|                            | O(6)        | $-x, -y, -z$                        | 3.166 (8)   |

| Atoms         | Angle        |
|---------------|--------------|
| O(2)–Cl–O(5)  | 152.5 (0.2)° |
| O(1)–Cl–O(4)  | 133.6 (0.2)  |
| O'(4)–Cl–O(6) | 158.0 (0.2)  |

| Atoms         | Angle       |
|---------------|-------------|
| O(1)–Cl–O(5)  | 75.4 (0.2)  |
| O(5)–Cl–O(4)  | 70.5 (0.2)  |
| O(4)–Cl–O(2)  | 110.1 (0.2) |
| O(2)–Cl–O(1)  | 114.6 (0.2) |
| O(1)–Cl–O'(4) | 81.0 (0.2)  |
| O'(4)–Cl–O(4) | 91.8 (0.2)  |
| O(4)–Cl–O(6)  | 100.3 (0.2) |
| O(6)–Cl–O(1)  | 103.0 (0.2) |
| O(2)–Cl–O'(4) | 84.9 (0.2)  |
| O'(4)–Cl–O(5) | 122.5 (0.2) |
| O(5)–Cl–O(6)  | 79.1 (0.2)  |
| O(6)–Cl–O(2)  | 73.7 (0.2)  |

### Discussion

The structure consists of corner-sharing  $[\text{Mg}^{2+} \cdot 6\text{H}_2\text{O}]$  and  $[\text{Cl}^- \cdot 6\text{H}_2\text{O}]$  octahedra as shown in Figs. 1, 2 and 3. The  $\text{Mg}^{2+}$  hydration polyhedra are centrosymmetric and regular with  $\text{Mg} \cdots \text{O}$  distances of  $2.062 \pm 0.003$  Å and  $\text{O} \cdots \text{O}$  edges of  $2.916 \pm 0.026$  Å. Within the experimental errors, these are perfect octahedra. In contrast, the  $\text{Cl}^-$  hydration polyhedra are very distorted

Table 3. *Bond distances and angles within the  $\text{Mg}(\text{H}_2\text{O})_6$  octahedron and between the corners of the octahedron and hydrogen-bonded neighbouring atoms*

Standard deviations are in parentheses referred to the last digits.

| From atom<br>in position<br>$(x, y, z)$ | to atoms | in positions                         | Distance    | Atoms           | Angle       |
|---|----------|--------------------------------------|-------------|-----------------|-------------|
| Mg                                      | O(1)     | $x, y, z$                            | 2.059 (7) Å | O(1)–Mg–O(2)    | 90.4 (0.3)° |
|   | O(2)     | $x, y, z$                            | 2.061 (7)   | O(1)–Mg–O(3)    | 88.5 (0.3)  |
|   | O(3)     | $x, y, z$                            | 2.065 (7)   | O(2)–Mg–O(3)    | 91.2 (0.3)  |
| O(1)                                    | Cl'      | $x, \frac{1}{2}-y, -1+\frac{1}{2}+z$ | 3.230 (7)   | Cl'–O(1)–O(5)   | 102.6 (0.3) |
|   | O(5)     | $1-x, \frac{1}{2}+y, \frac{1}{2}-z$  | 2.811 (10)  |                 |             |
| O(2)                                    | Cl       | $x, y, z$                            | 3.261 (8)   | Cl–O(2)–O(6)    | 98.5 (0.3)  |
|   | O(6)     | $-x, \frac{1}{2}+y, \frac{1}{2}-z$   | 2.721 (10)  |                 |             |
| O(3)                                    | O(4)     | $x, y, z$                            | 2.779 (10)  | O(4)–O(3)–O'(5) | 103.2 (0.3) |
|   | O'(5)    | $x, -1+\frac{1}{2}-y, \frac{1}{2}+z$ | 2.805 (10)  |                 |             |

(Fig. 4). The  $\text{Cl}^- \cdots \text{O}$  distances range from 3.11 Å to 3.26 Å and the  $\text{O} \cdots \text{O}$  edges from 3.86 Å to 5.55 Å. There are correspondingly wide deviations from orthogonal coordination angles at the chlorine, as shown in Table 3.

The hydration octahedra share corners so as to form layers which extend parallel to (100). These layers are shown most clearly, separated by zigzag channels, in projection on (010) on the right hand side of Fig. 1. The layers of corner-linked octahedra are then hydrogen-bonded together between corners and across the channels as indicated by the heavy lines on the left hand side of the figure. The hydrogen bonds form links between the corners of anionic and cationic octahedra, and pairs of anionic octahedra, but not between pairs of cationic octahedra. It can also be seen in Fig. 1 that all like octahedra form chains in the  $c$ -axis direction with the  $\text{Cl}^-$  octahedra sharing a common corner.

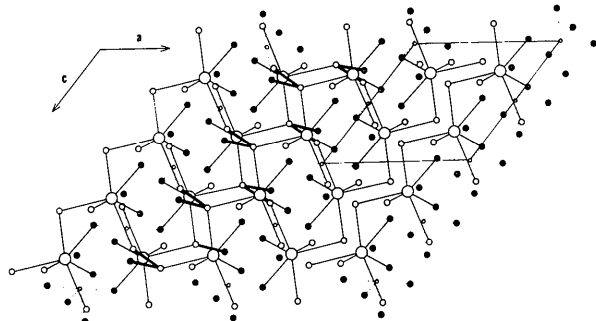


Fig. 1. The structure of magnesium dodecahydrate in orthogonal projection on the (100) plane. Smallest open circles,  $\text{Mg}^{2+}$ ; filled circles, water oxygen belonging to magnesium octahedron; large open circles,  $\text{Cl}^-$ ; medium open circles, water oxygen belonging to chloride octahedron. The magnesium and chloride octahedral coordinations are indicated by dotted and full lines respectively. Zigzag channels separate the layers of corner-sharing octahedra. The heavy lines across these channels are the hydrogen bonds between the layers.

$\text{Mg}^{2+}$  octahedra are not directly linked, but are invariably interspersed by those of the anions, as shown in Figs. 2 and 3. They are, in fact, as removed from each other in the structure as possible. The distortions of the  $[\text{Cl}^- \cdot 6\text{H}_2\text{O}]$  from octahedral symmetry are unsymmetrical and each individual polyhedron is enantiomorphic, so that left- and right-handedly distorted octahedra alternate along the corner-sharing chain in the  $c$ -axis direction. This is a consequence of the particular arrangement of the polyhedra, as seen by consideration of Fig. 2, for example.

If it is assumed that each water molecule in the structure forms two hydrogen bonds and that these are directed either toward the anions or toward other

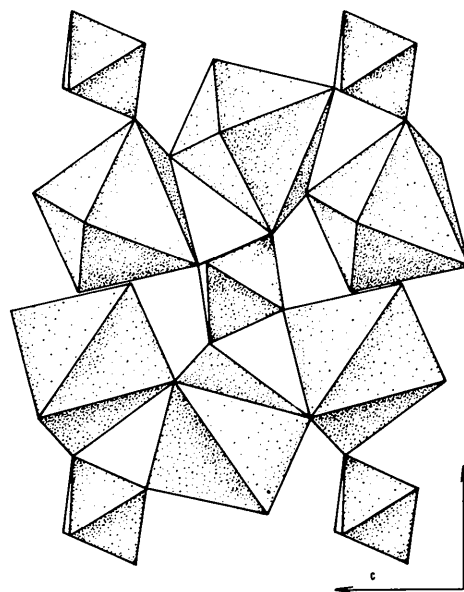


Fig. 2. The corner-sharing magnesium and chloride hydration octahedra which form the layer of the structure parallel to (100), in orthogonal projection on the (100) plane.

Table 5. *Hydrogen-bond distances and angles of the water molecules*

The angles are calculated assuming linear hydrogen bonds.

| Central atom in ( $x, y, z$ ) | to atom forming tetrahedron | in positions                                | Distance    | Atoms          | Angle | $\sigma$ |
|-------------------------------|-----------------------------|---|-------------|----------------|-------|----------|
| O(4)                          | Cl                          | $1-x, -y, 1-z$                              | 3.112 (8) Å | Cl—O(4)—Cl'    | 96.6° | 0.2°     |
|                               | Cl'                         | $1-x, -1+(\frac{1}{2}+y), \frac{1}{2}-z$    | 3.186 (8)   | Cl—O(4)—O(3)   | 124.2 | 0.3      |
|                               | O(3)                        | $x, y, z$                                   | 2.779 (11)  | Cl—O(4)—O(6)   | 94.3  | 0.3      |
|                               | O(6)                        | $x, y, z$                                   | 2.758 (11)  | Cl'—O(4)—O(3)  | 117.8 | 0.3      |
|                               |                             |   |             | Cl'—O(4)—O(6)  | 123.1 | 0.3      |
|                               |                             |   |             | O(3)—O(4)—O(6) | 100.4 | 0.3      |
| O(5)                          | Cl                          | $x, -1+y, z$                                | 3.211 (7)   | Cl—O(5)—O(1)   | 106.4 | 0.3      |
|                               | O(1)                        | $1-x, -1+(\frac{1}{2}+y), \frac{1}{2}-z$    | 2.811 (10)  | Cl—O(5)—O(3)   | 120.2 | 0.3      |
|                               | O(3)                        | $x, -1+(\frac{1}{2}-y), -1+(\frac{1}{2}+z)$ | 2.805 (10)  | Cl—O(5)—O(6)   | 113.1 | 0.3      |
|                               | O(6)                        | $x, y, z$                                   | 2.752 (10)  | O(1)—O(5)—O(3) | 116.9 | 0.3      |
|                               |                             |   |             | O(1)—O(5)—O(6) | 104.1 | 0.3      |
|                               |                             |   |             | O(3)—O(5)—O(6) | 94.7  | 0.3      |
| O(6)                          | Cl                          | $-x, -y, -z$                                | 3.166 (8)   | Cl—O(6)—O(2)   | 100.7 | 0.3      |
|                               | O(2)                        | $-x, -1+(\frac{1}{2}+y), \frac{1}{2}-z$     | 2.721 (10)  | Cl—O(6)—O(5)   | 108.9 | 0.3      |
|                               | O(5)                        | $x, y, z$                                   | 2.752 (10)  | Cl—O(6)—O(4)   | 116.1 | 0.3      |
|                               | O(4)                        | $x, y, z$                                   | 2.758 (11)  | O(2)—O(6)—O(5) | 118.9 | 0.3      |
|                               |                             |   |             | O(2)—O(6)—O(4) | 113.7 | 0.3      |
|                               |                             |   |             | O(5)—O(6)—O(4) | 99.3  | 0.3      |

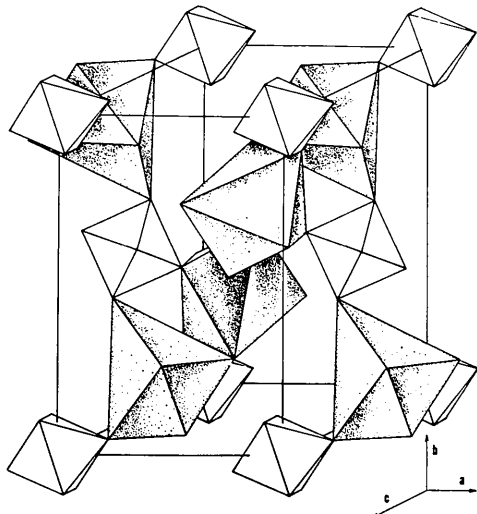


Fig. 3. The unit cell of magnesium chloride dodecahydrate, viewed in perspective, with two additional chloride octahedra at the top of left and bottom of right sides. The small octahedra are  $[\text{Mg}^+ \cdot 6\text{H}_2\text{O}]$  and the large octahedra are  $[\text{Cl}^- \cdot 6\text{H}_2\text{O}]$ .

water molecules, every proton in the structure can be associated with a hydrogen-bond distance and its donor/acceptor position can be uniquely predicted. This hydrogen-bond distribution and proton assignment is represented diagrammatically below.

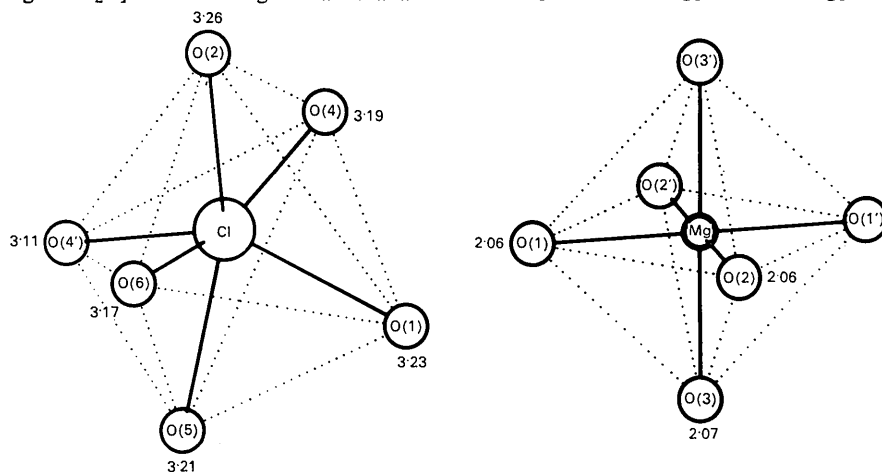
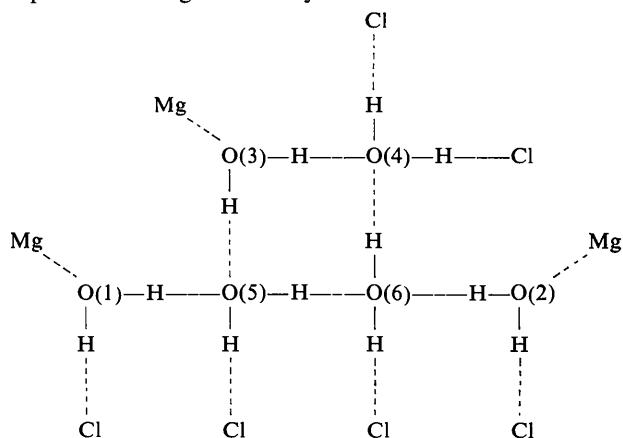


Fig. 4. The distances of the water molecules from the central ions in the hydration polyhedra.

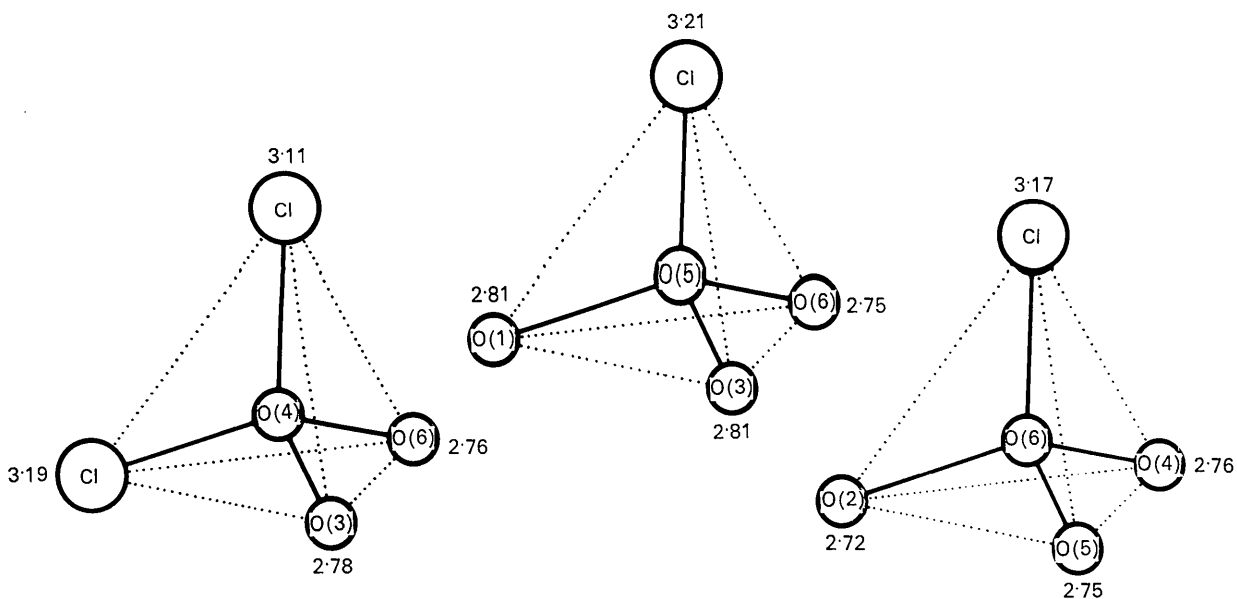


Fig. 5. The tetrahedral environment of the three four-coordinated water molecules with their hydrogen-bond distances.

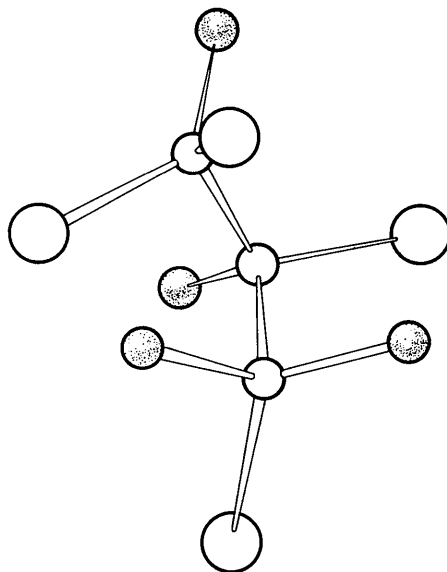


Fig. 6. The orthogonal projection of the three water molecules O(4), O(5) and O(6) with their tetrahedrally coordinated neighbours. The large spheres are chloride ions and the small ones are the water molecules. The dotted spheres are water molecules belonging to magnesium octahedra.

It is probable that the hydrogen atoms do not lie very far from the line of centers of the heavier atoms, since the hydrogen-bond angles, assuming linearity of the bond, do not deviate by more than  $16^\circ$  from tetrahedral. This approximately tetrahedral environment of the

three four-coordinated water molecules, O(4), O(5) and O(6) is illustrated in Figs. 5 and 6.

This is clearly a very reasonable structure for a hydrated phase in which the stoichiometry is such that each ion can have a complete hydration polyhedron and there is no direct anion-cation contact. It is interesting to note that all the distortions from regularity necessary to form this pattern of associated octahedra are to be found in the hydrogen-bonded  $[\text{Cl}^- \cdot 6\text{H}_2\text{O}]$  octahedra, and that this distortion has an enantiomorphic sense even though the whole crystal structure is centrosymmetrical. The more tightly bound ionic  $[\text{Mg}^+ \cdot 6\text{H}_2\text{O}]$  octahedra are completely undistorted within the accuracy of the structure determination.

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#### References

- ALLEN, K. W., JEFFREY, G. A. & McMULLAN, R. K. (1963). *Rev. Sci. Instrum.* **34**, 488.  
 ANDRESS, K. R. & GUNDERMANN, J. (1934). *Z. Kristallogr.* **A 87**, 345.  
 BEURSKENS, P. T. (1964). *Acta Cryst.* **17**, 462.  
 BUSING, W. R., MARTIN, K. O. & LEVY, H. A. (1962). *A Fortran Crystallographic Least Squares Refinement Program for the IBM 7090*. ORNL-TM-305. Oak Ridge National Laboratory, Tennessee.  
 PANFILOW, J. (1894). *J. Russ. Ges. [chem]*, **26**, 234.  
 HOFF, J. H. VAN'T & MEYERHOFFER, W. (1898). *Z. Phys. Chem.* **27**, 75.

*Acta Cryst.* (1966). **20**, 881

## The Evaluation of Phases for Structure Determination by Neutron Diffraction

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The formulas for phase determination often used for X-ray diffraction data are not immediately applicable to neutron diffraction data when atoms with both positive and negative scattering factors are present. A formula is presented from which normalized structure factors for the squared structure may be calculated from the normalized scattering data, as obtained from a neutron diffraction experiment. Since the squared structure is defined as the structure which scatters with the square of the scattering factor for each atom in the original structure, the computed structure factors for the squared structure will always represent a positive structure and the formulas for phase determination used for X-ray data will be applicable. Test calculations are presented.

### Introduction

The positions of the atoms in the unit cell of a crystal can be calculated by Fourier series methods, provided that both the amplitudes and phases of the scattered neutron waves are known. The amplitudes and phases

form the coefficients of a Fourier series representing the neutron scattering density in the unit cell, and the main maxima of such a function correspond to the positions of the atomic nuclei. Fourier series methods have been employed for several years starting with the investigations of Bacon & Pease (1953) and Peterson, Levy &